

Here I summarize the work done in studying the direct deconvolution method: in section one, the theory behind this method is presented; in section two, the results of different tests done with the method that have been obtained; in section three, the conclusions that are drawn from these tests; and in section four, some open issues still waiting for interpretation together with upcoming results of new analysis.

Section one: Theory behind the Direct Deconvolution.

Defining X as a continuous function describing the light variation of a pulsating star then, the product $X \cdot W$ is the observed light variation denoted by

$$f_w(X) = X \cdot W \quad (1)$$

Where W is the window function, which can be extended to the concept of a effective window if we introduce any instrumental response as a function of we call I_e so giving the definition of the effective window W_e

$$W_e = W \cdot I_e \quad (2)$$

In such a way, this definition give us a clear independence between the window and the function X .

If this window is in general not evenly-spaced then, we can apply the Fourier Transform for unevenly spaced data, here denoted by $FT_{ls}[\cdot]$, which is the Lomb-Scargle estimation of the Fourier Transform (see Scargle 1989 for details). So, this function is the unbiased estimation of the Fourier Transform $FT[\cdot]$

$$FT_{ls}[f_w(X)] = FT_{ls}[X \cdot W] = FT[X] * FT_{ls}[W] \quad (2)$$

Where dot means product and asterisk convolution.

Note that X does not depend on the observational window.

Taking the Fast Fourier Transform of (2) denoted by $fft\{\cdot\}$,

$$fft\{FT_{ls}[f_w(X)]\} = fft\{FT[X] * FT_{ls}[W]\} = fft\{FT[X]\} \cdot fft\{FT_{ls}[W]\} \quad (3)$$

Now is possible to obtain a ratio because the $fft\{FT_{ls}[W]\}$ is a complex function of the Lomb-Scargle Fourier Transform of the window, which is now evenly sampled in frequency following prescriptions established in Scargle (1989).

Then,

$$\frac{fft\{FT_{ls}[f_w(X)]\}}{fft\{FT_{ls}[W] + \epsilon\}} = fft\{FT[X]\} \quad (4)$$

Where ϵ stands for numerical stability, some test are performed to study its contribution to (4).

Finally, taking the Inverse Fast Fourier Transform, $fft^{-1}[\cdot]$, we will obtain the 'true' Fourier Transform of the signal without any dependence of the window function.

$$fft^{-1}\left[\frac{fft\{FT_{ls}[f_w(X)]\}}{fft\{FT_{ls}[W]\}}\right] = fft^{-1}\{fft\{FT[X]\}\} = FT[X] \quad (5)$$

Which is, by construction, clearly independent of the window.

Section two:Results.

In the next section, we present some results based on the theory explained in section two, applied to real data from space satellites such as CoRoT and Kepler, where in the case of CoRoT, a periodic window is hampering the data and in the case of Kepler, the space between observation is not strictly even distributed.

a)Effects of ϵ in (4):

As shown in Figure1 and using the HD174936 data, we can say that the Direct Deconvolution method (DD) here presented does not have a big dependency on the choice of ϵ , only a very small difference in power are obtained when a very different value of ϵ is selected(ranging from 0.001 to 25).

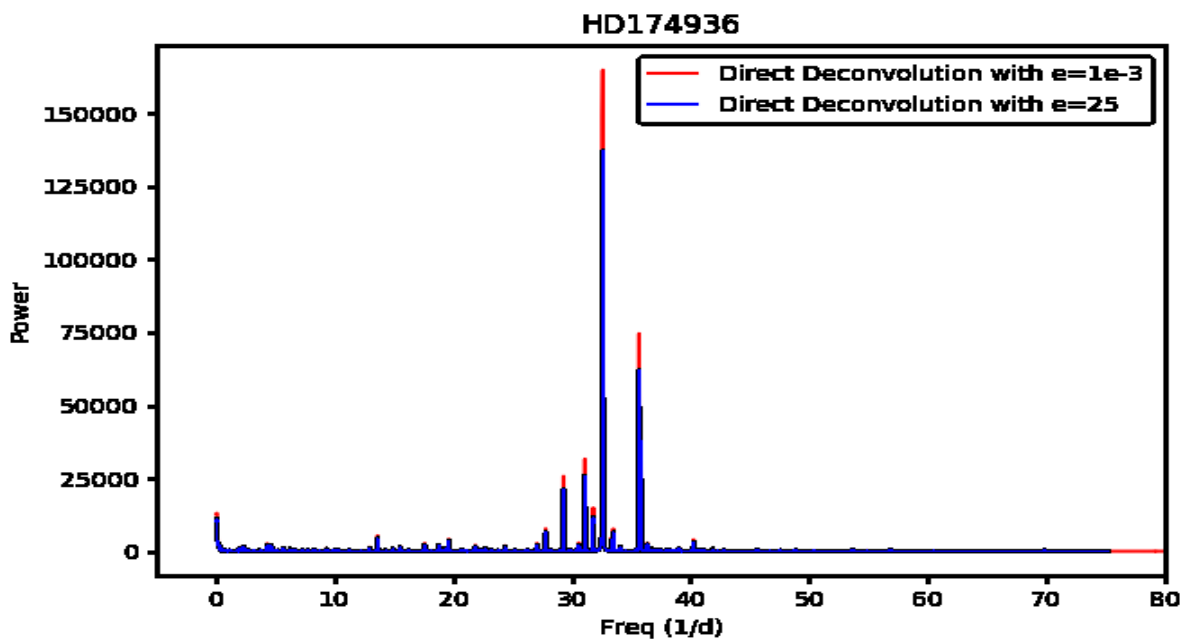


Figure1:The power change a small amount when the ϵ is a few orders of magnitude bigger.

b)Instrumental efficiency W_e as a window:

The last plot show a DD of a signal where the window function was a box of length n . Now, taking I_e as the light variations of an assumed constant star, observed in the same conditions as the target HD174936 we obtained the periodogram shown in Figure2 . As can be seen in Figure2, and compared with Figure1 and other targets in the next sections, there is no gain in s/n ratio when we use an effective window built as an assumed constant star.

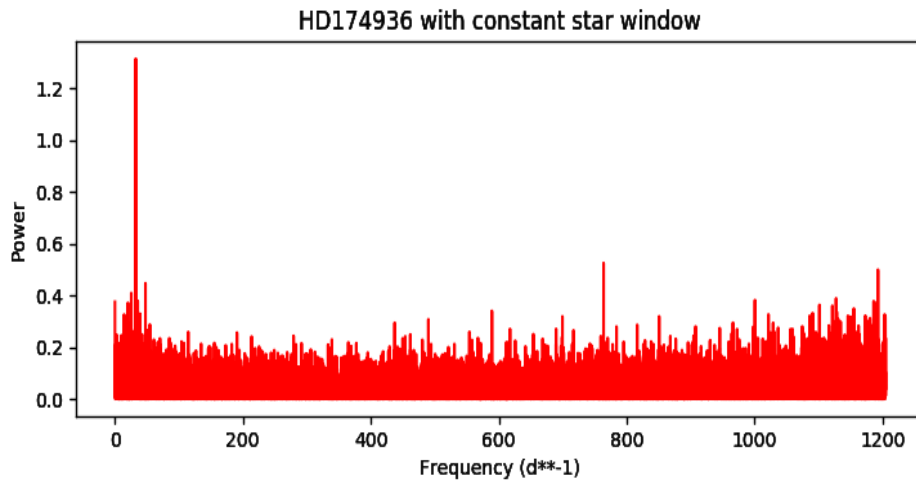


Figure2: Direct Deconvolution using a constant star (HD175542) of the same run as an effective window function.

c) Beyond Nyquist:

The choice of the $f_{\text{m}\acute{\text{a}}\text{x}}$, when dealing with unevenly-space data, can be done in many ways, and is an open issue when dealing with this type of data.

A test was performed for different $f_{\text{m}\acute{\text{a}}\text{x}}$, taking multiples of the fundamental frequency. This is shown in Figure4 for the High Amplitude Delta Scuti star (HADS) observed by Kepler KIC5950759.

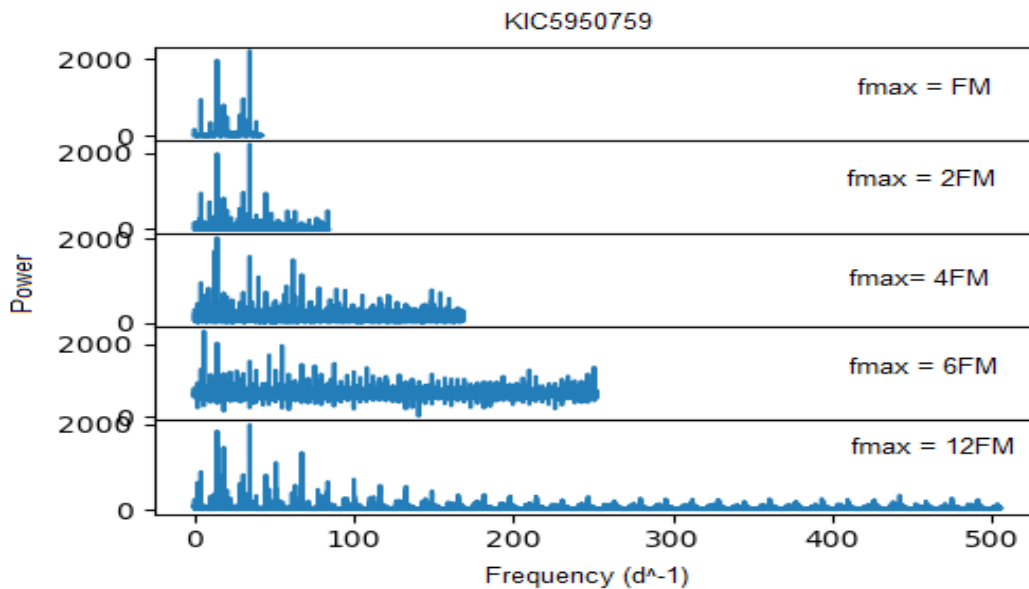


Figure3: Different $f_{\text{m}\acute{\text{a}}\text{x}}$, taken it as a multiple of the fundamental frequency F_0 .
Where $F_m = (m-1)*F_0$, and $m = 2^{(n)}$ being n the number of data points .

From the many choices we can make for a numeric value of $f_{\text{m}\acute{\text{a}}\text{x}}$ found in the literature for unevenly-spaced data, that of the Eyer & Bartholdi definition (Eyer & Bartholdi, 1999) is the mathematically proven Nyquist limit of a time series unevenly sampled, but is frequently computationally intractable.

In order to see what happen beyond usual definitions of this frequency we have made an extension up to $500d^{-1}$, (which is only the 0,01% of Eyer's limit!!!), showing that in this

condition, the power spectrum is a clear repetition as predicted by the classical theory. In any case, in Murphy (2013), these issues are discussed in detail for the particular case of Kepler.

d) *Kepler data:*

In order to check the possibilities of this method, we start applying the pipeline in the Appendix1 to Kepler data.

An example is shown in the Figure6, using the HADS KIC5950759.

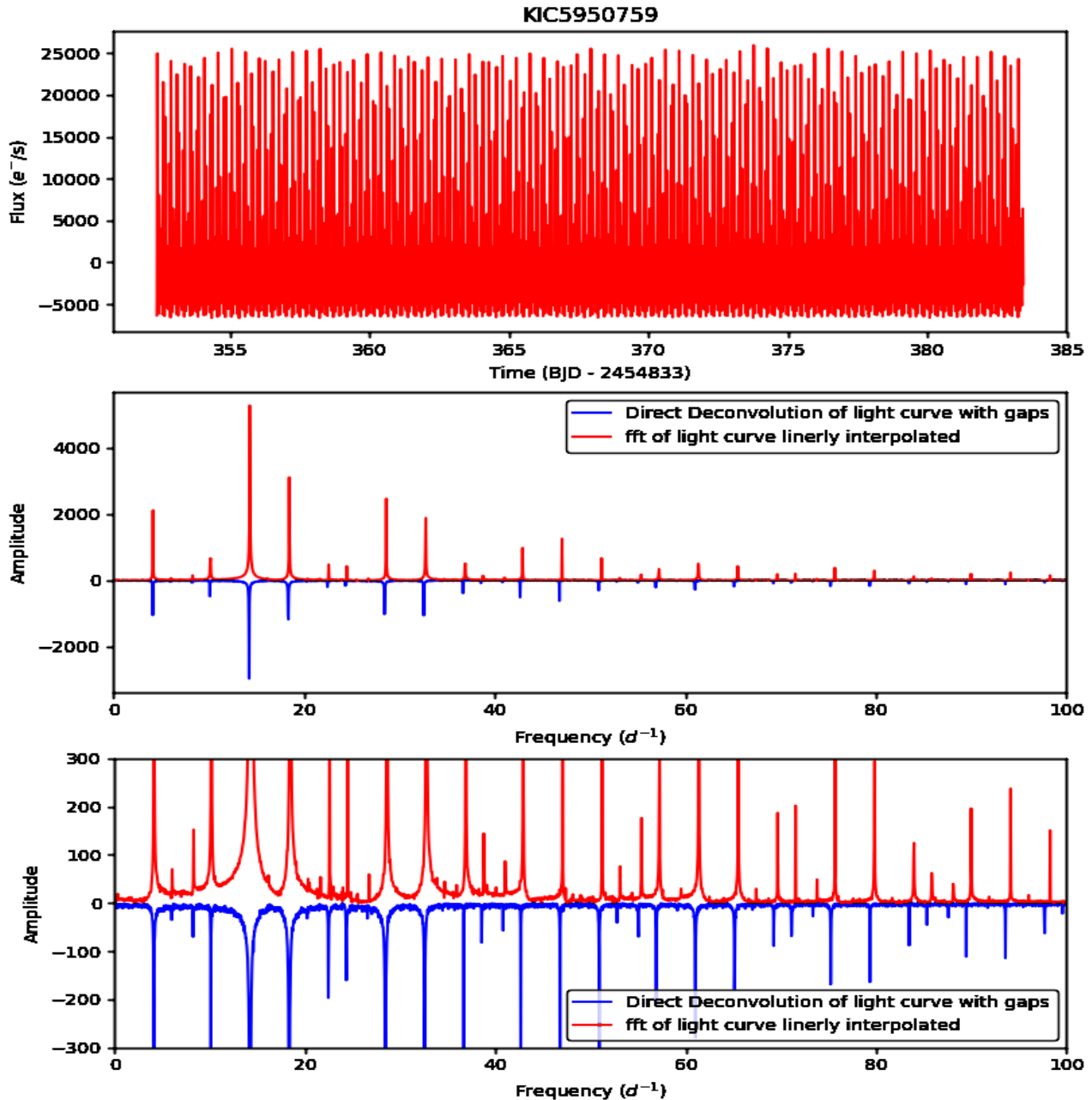


Figure4: Upper panel: KIC5950759 light curve.
Mid panel: Direct Deconvolution vs fft.
Lower panel: Zoom in amplitude.

The idea to apply the method to this star is that we expect the same power spectra as the calculated by a fft routine because the window of the observations made by Kepler is essentially the classical product with a Dirac delta and a box, except for a few points (around 190) that were filled by a linear interpolation. However, there is a small frequency shift in the fft plot with respect to Direct Deconvolution's, in the sense that the frequency obtained by the Direct Deconvolution is smaller than the obtained by the fft. This could be originated by the non-exactly constant cadence that Kepler satellite pipeline give after the Barycentric time stamp correction.

All these numerical results can be summarize in Table1 (1.1 ordered by frequencies and 1.2 ordered by amplitude). These frequencies have been selected by using the Fisher test for periodicities equivalent to s/n ratio of about 50 for fft, and 51 for DD.

Table1.1: Frequencies as given by different methods for KIC5950759.

fft (lc linearly interpolated)		Direct Deconvolution		SigSpec's prewhitening	
Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude
4.1016	334.7625	4.0891	204.9595	4.1159	2173.6718
4.1466	2115.3518	4.1213	1046.6558	5.9899	59.0632
4.1917	277.2112	4.1535	160.625	8.2316	155.5602
10.1412	576.317	10.11	479.069	10.1055	978.5853
10.1863	668.7678	14.0058	141.4078	14.2214	6942.8012
13.9273	261.8616	14.038	158.6087	14.5401	75.1634
13.9724	304.2558	14.0702	197.299	18.3372	3490.8824
14.0174	328.9283	14.1024	247.9556	20.211	66.1817
14.0625	392.887	14.1346	338.5197	22.453	495.7548
14.1076	483.0523	14.1668	539.5061	24.3269	430.1171
14.1526	629.7198	14.199	1308.0868	26.5671	56.1843
14.1977	883.2617	14.2312	2958.4864	28.4427	2631.6651
14.2428	1505.6777	14.2634	696.3058	28.762	53.8044
14.2879	5271.3633	14.2956	387.0512	32.5586	2289.6589
14.3329	3488.2517	14.3278	269.7717	36.6744	769.51
14.378	1302.3374	14.36	205.3842	38.5484	188.9258
14.4231	801.9923	14.3922	165.5433	40.7895	112.5227
14.4681	576.5599	14.4244	138.1833	42.6641	1035.2667
14.5132	450.8523	18.2237	151.6935	46.78	1257.596
14.5583	365.3517	18.2559	215.6184	50.8958	673.635
14.6034	311.0315	18.2881	354.0978	52.7697	76.2612
14.6484	265.3469	18.3203	1049.5386	55.0114	193.0238
18.3444	342.0793	18.3525	1174.3673	56.8855	462.0195
18.3894	622.4033	18.3847	382.3912	61.0014	725.4212
18.4345	3105.2348	18.4169	227.6978	65.1172	506.4338
18.4796	1097.8073	18.4491	169.5445	69.233	196.8085
18.5246	478.3108	18.4812	126.1553	71.1069	204.7195
18.5697	322.5664	22.4415	196.5924	73.3482	50.3374
22.5811	478.349	24.3412	160.119	75.2228	395.8955
24.4742	429.119	28.3658	158.4686	79.3386	340.352
28.5306	267.0257	28.398	291.3719	83.4544	173.1713
28.5757	617.6771	28.4302	1015.6121	85.3282	86.1301
28.6208	2464.6742	28.4624	639.1926	87.5697	56.518
28.6659	418.4761	28.4946	240.3643	89.4441	222.3444
32.6773	373.9501	28.5268	149.3246	93.5599	243.6207
32.7224	965.2707	32.5193	192.178	97.6757	151.3599
32.7674	1882.3565	32.5515	1054.7149	99.5498	44.1687
32.8125	491.5365	32.5837	297.0216	101.7913	61.935
32.8576	293.3165	32.6159	131.2211	103.6654	132.3102
36.869	465.6416	36.6727	383.3521	107.7813	174.2141
36.9141	512.118	42.6614	510.7592	111.8971	125.2499
42.9087	980.093	46.7827	615.8533	116.0129	61.488
47.0553	1258.5049	50.9039	302.4202	117.8868	80.2827
51.2019	670.1399	56.8926	211.6253	122.0027	122.9203
57.2416	343.5094	60.9817	177.6353	126.1185	102.5781
61.3431	505.6594	61.0139	278.698	130.2342	56.8933
61.3882	414.4852	65.1029	180.3176	132.1081	48.9599
65.4898	429.696	65.1351	143.3025	136.224	83.2783
75.6761	380.1309	75.2129	168.6472	140.3399	80.6606
79.8227	294.7806	79.3341	163.9653	144.4557	50.8893

The frequency resolution is $1/(27,18889 \text{ d}) = 0,04(\text{d}^{-1})$

Table1.2: Frequencies as given by different methods for KIC5950759.

fft(linerly interpolated)		Direct Deconvolution		SigSpec's prewhitening	
Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude
14.2879	5271.3633	14.2312	2958.4864	14.2214	6942.8012
14.3329	3488.2517	14.1990	1308.0868	18.3372	3490.8824
18.4345	3105.2348	18.3525	1174.3673	28.4427	2631.6651
28.6208	2464.6742	32.5515	1054.7149	32.5586	2289.6589
4.1466	2115.3518	18.3203	1049.5386	4.1159	2173.6718
32.7674	1882.3565	4.1213	1046.6558	46.7800	1257.5960
14.2428	1505.6777	28.4302	1015.6121	42.6641	1035.2667
14.3780	1302.3374	14.2634	696.3058	10.1055	978.5853
47.0553	1258.5049	28.4624	639.1926	36.6744	769.5100
18.4796	1097.8073	46.7827	615.8533	61.0014	725.4212
42.9087	980.0930	14.1668	539.5061	50.8958	673.6350
32.7224	965.2707	42.6614	510.7592	65.1172	506.4338
14.1977	883.2617	10.1100	479.0690	22.4530	495.7548
14.4231	801.9923	14.2956	387.0512	56.8855	462.0195
51.2019	670.1399	36.6727	383.3521	24.3269	430.1171
10.1863	668.7678	18.3847	382.3912	75.2228	395.8955
14.1526	629.7198	18.2881	354.0978	79.3386	340.3520
18.3894	622.4033	14.1346	338.5197	93.5599	243.6207
28.5757	617.6771	50.9039	302.4202	89.4441	222.3444
14.4681	576.5599	32.5837	297.0216	71.1069	204.7195
10.1412	576.3170	28.3980	291.3719	69.2330	196.8085
36.9141	512.1180	61.0139	278.6980	55.0114	193.0238
61.3431	505.6594	14.3278	269.7717	38.5484	188.9258
32.8125	491.5365	14.1024	247.9556	107.7813	174.2141
14.1076	483.0523	28.4946	240.3643	83.4544	173.1713
22.5811	478.3490	18.4169	227.6978	8.2316	155.5602
18.5246	478.3108	18.2559	215.6184	97.6757	151.3599
36.8690	465.6416	56.8926	211.6253	103.6654	132.3102
14.5132	450.8523	14.3600	205.3842	111.8971	125.2499
65.4898	429.6960	4.0891	204.9595	122.0027	122.9203
24.4742	429.1190	14.0702	197.2990	40.7895	112.5227
28.6659	418.4761	22.4415	196.5924	126.1185	102.5781
61.3882	414.4852	32.5193	192.1780	85.3282	86.1301
14.0625	392.8870	65.1029	180.3176	136.2240	83.2783
75.6761	380.1309	60.9817	177.6353	140.3399	80.6606
32.6773	373.9501	18.4491	169.5445	117.8868	80.2827
14.5583	365.3517	75.2129	168.6472	52.7697	76.2612
57.2416	343.5094	14.3922	165.5433	14.5401	75.1634
18.3444	342.0793	79.3341	163.9653	20.2110	66.1817
4.1016	334.7625	4.1535	160.6250	101.7913	61.9350
14.0174	328.9283	24.3412	160.1190	116.0129	61.4880
18.5697	322.5664	14.0380	158.6087	5.9899	59.0632
14.6034	311.0315	28.3658	158.4686	130.2342	56.8933
13.9724	304.2558	18.2237	151.6935	87.5697	56.5180
79.8227	294.7806	28.5268	149.3246	26.5671	56.1843
32.8576	293.3165	65.1351	143.3025	28.7620	53.8044
4.1917	277.2112	14.0058	141.4078	144.4557	50.8893
28.5306	267.0257	14.4244	138.1833	73.3482	50.3374
14.6484	265.3469	32.6159	131.2211	132.1081	48.9599
13.9273	261.8616	18.4812	126.1553	99.5498	44.1687

The frequency resolution is $1/(27,18889 \text{ d}) = 0,04(\text{d}^{-1})$

d) Comparisons with MIARMA and Inpainting:

In order to investigate what is the impact on more critical window as the periodic window in the CoRoT data, a similar pipeline has been applied to the CoRoT stars HD174936 and HD174966.

Table 2.1: Frequencies as given by different methods for HD174936.

fft (inpainting interpolated)		fft(MIARMA interpolated)		Direct Deconvolution		SigSpec's prewhitening	
Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude
0.0412	220.5501	0.0412	112.7400	0.0735	112.6624	0.05703166	295
0.1236	70.0460	0.0824	127.6567	0.1103	114.8908	0.10902264	151
2.1835	83.9824	2.1835	85.1341	0.1471	77.6395	1.89881897	106
2.3071	74.0755	2.3071	75.0439	0.1839	56.5957	2.13465952	93
4.3259	130.5756	4.3259	136.4733	0.2206	52.0447	2.19027978	153
13.6368	124.1881	13.6368	120.1615	0.2574	49.4258	2.30263739	112
13.6780	81.5247	13.6780	85.5493	0.2942	46.5516	2.39270087	106
15.5731	76.7233	15.5731	81.4821	0.3677	44.8741	4.32305059	234
18.8278	73.6539	21.8765	99.9206	2.3902	42.8883	4.37191456	87
19.2810	73.3452	24.3896	107.4802	4.3023	43.6812	4.81579927	107
21.8765	98.3252	27.0676	99.7160	4.3391	51.2824	11.23851905	99
24.3896	101.0537	27.8091	220.7062	4.6700	45.9662	13.65217685	276
27.0676	99.7860	28.2623	74.0611	13.6423	72.6177	15.56429237	146
27.8091	210.0884	29.2923	260.2221	17.6136	52.1699	18.82635651	118
28.2623	75.8035	29.3335	159.4579	18.8271	47.5923	19.29104143	143
29.2923	238.5662	30.4871	74.9975	19.6361	63.3588	21.89159010	208
29.3335	159.8790	31.0638	214.3807	21.8791	46.2056	24.34752268	94
31.0638	205.1459	31.1050	350.7074	24.3796	43.3906	24.39438984	172
31.1050	338.5683	31.7642	120.3859	27.0639	51.8003	26.20918771	91
31.7642	118.3908	31.8054	271.3979	27.7994	88.7974	27.05351826	223
31.8054	257.6611	31.8466	90.4482	27.8361	42.1390	27.81356301	362
31.8466	83.0270	32.4234	75.5156	29.3070	160.8512	28.25829334	135
32.4234	72.7873	32.4646	94.7881	30.5940	55.1842	29.30857423	546
32.4646	93.3642	32.5058	139.5810	31.0353	42.7069	29.84734370	91
32.5058	132.9238	32.5470	241.4739	31.0721	70.5522	30.50386759	170
32.5470	233.1724	32.5882	1147.3994	31.1088	177.9488	31.05789316	300
32.5882	1102.4011	32.6294	379.9658	31.7707	73.8471	31.11058413	565
32.6294	363.3899	32.6706	168.6981	31.8075	122.1545	31.20382877	100
32.6706	160.0924	32.7118	94.7551	31.8443	50.4966	31.79201646	530
32.7118	91.3109	32.7530	81.5987	32.4326	44.2821	32.59856949	2122
32.7530	77.1043	33.3298	155.2867	32.4694	54.8353	33.34080444	291
33.3298	155.9896	33.4946	189.1406	32.5061	76.3519	33.49080418	319
33.4946	178.7411	34.1125	111.1794	32.5429	132.0195	34.10273802	187
34.1125	100.6263	35.3485	78.2449	32.5797	386.3938	35.36087443	89
35.3485	73.3993	35.5545	109.1304	32.6165	406.3982	35.52946280	112
35.5545	99.9943	35.5957	129.6492	32.6532	138.6244	35.62455049	96
35.5957	119.7450	35.6369	381.6082	32.6900	80.7681	35.65822312	1016
35.6369	361.1428	35.6781	409.1384	32.7268	58.7035	35.82316189	716
35.6781	393.7703	35.7193	137.5606	32.7635	40.7946	36.37860618	216
35.7193	129.7681	35.7605	133.7600	33.3151	43.4446	36.75409664	162
35.7605	131.9806	35.8017	258.8149	33.3519	68.6059	37.48735055	113
35.8017	257.5258	35.8429	292.1253	33.4990	87.9903	37.85748889	103
35.8429	274.4931	35.8841	106.1599	34.0873	41.9818	38.18628638	127
35.8841	98.2385	35.9253	98.5931	35.6317	105.8611	38.93299170	93
35.9253	87.9843	36.3785	133.1636	35.6685	273.3277	39.07015345	174
36.3785	130.5713	36.7493	105.8829	35.7053	75.9357	40.18442027	98
36.7493	102.5880	37.4908	76.7573	35.7788	51.9179	40.25335199	241
38.1912	74.3104	38.1912	82.4883	35.8156	212.4741	45.71460589	98
39.0564	86.0510	39.0564	97.1459	36.3672	52.9667	48.87780781	76
40.2512	136.0316	40.2512	149.3952	40.2650	63.5834	53.68998252	98

The frequency resolution is $1/(27,19666 \text{ d}) = 0,04(\text{d}^{-1})$

Table2.2: Frequencies as given by different methods for HD174936 (amplitude ordered).

fft (inpainting interpolated)		fft(MIARMA interpolated)		Direct Deconvolution		SigSpec's prew hitening	
Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude
32.5882	1102.4011	32.5882	1147.3994	32.6165	406.3982	32.59856949	2122
35.6781	393.7703	35.6781	409.1384	32.5797	386.3938	35.65822312	1016
32.6294	363.3899	35.6369	381.6082	35.6685	273.3277	35.82316189	716
35.6369	361.1428	32.6294	379.9658	35.8156	212.4741	31.11058413	565
31.1050	338.5683	31.1050	350.7074	31.1088	177.9488	29.30857423	546
35.8429	274.4931	35.8429	292.1253	29.3070	160.8512	31.79201646	530
31.8054	257.6611	31.8054	271.3979	32.6532	138.6244	27.81356301	362
35.8017	257.5258	29.2923	260.2221	32.5429	132.0195	33.49080418	319
29.2923	238.5662	35.8017	258.8149	31.8075	122.1545	31.05789316	300
32.5470	233.1724	32.5470	241.4739	0.1103	114.8908	0.05703166	295
0.0412	220.5501	27.8091	220.7062	0.0735	112.6624	33.34080444	291
27.8091	210.0884	31.0638	214.3807	35.6317	105.8611	13.65217685	276
31.0638	205.1459	33.4946	189.1406	27.7994	88.7974	40.25335199	241
33.4946	178.7411	32.6706	168.6981	33.4990	87.9903	4.32305059	234
32.6706	160.0924	29.3335	159.4579	32.6900	80.7681	27.05351826	223
29.3335	159.8790	33.3298	155.2867	0.1471	77.6395	36.37860618	216
33.3298	155.9896	40.2512	149.3952	32.5061	76.3519	21.89159010	208
40.2512	136.0316	32.5058	139.5810	35.7053	75.9357	34.10273802	187
32.5058	132.9238	35.7193	137.5606	31.7707	73.8471	39.07015345	174
35.7605	131.9806	4.3259	136.4733	13.6423	72.6177	24.39438984	172
4.3259	130.5756	35.7605	133.7600	31.0721	70.5522	30.50386759	170
36.3785	130.5713	36.3785	133.1636	33.3519	68.6059	36.75409664	162
35.7193	129.7681	35.5957	129.6492	40.2650	63.5834	2.19027978	153
13.6368	124.1881	0.0824	127.6567	19.6361	63.3588	0.10902264	151
35.5957	119.7450	31.7642	120.3859	32.7268	58.7035	15.56429237	146
31.7642	118.3908	13.6368	120.1615	0.1839	56.5957	19.29104143	143
36.7493	102.5880	0.0412	112.7400	30.5940	55.1842	28.25829334	135
24.3896	101.0537	34.1125	111.1794	32.4694	54.8353	38.18628638	127
34.1125	100.6263	35.5545	109.1304	36.3672	52.9667	18.82635651	118
35.5545	99.9943	24.3896	107.4802	17.6136	52.1699	37.48735055	113
27.0676	99.7860	35.8841	106.1599	0.2206	52.0447	35.52946280	112
21.8765	98.3252	36.7493	105.8829	35.7788	51.9179	2.30263739	112
35.8841	98.2385	21.8765	99.9206	27.0639	51.8003	4.81579927	107
32.4646	93.3642	27.0676	99.7160	4.3391	51.2824	2.39270087	106
32.7118	91.3109	35.9253	98.5931	31.8443	50.4966	1.89881897	106
35.9253	87.9843	39.0564	97.1459	0.2574	49.4258	37.85748889	103
39.0564	86.0510	32.4646	94.7881	18.8271	47.5923	31.20382877	100
2.1835	83.9824	32.7118	94.7551	0.2942	46.5516	11.23851905	99
31.8466	83.0270	31.8466	90.4482	21.8791	46.2056	53.68998252	98
13.6780	81.5247	13.6780	85.5493	4.6700	45.9662	45.71460589	98
32.7530	77.1043	2.1835	85.1341	0.3677	44.8741	40.18442027	98
15.5731	76.7233	38.1912	82.4883	32.4326	44.2821	35.62455049	96
28.2623	75.8035	32.7530	81.5987	4.3023	43.6812	24.34752268	94
38.1912	74.3104	15.5731	81.4821	33.3151	43.4446	38.93299170	93
2.3071	74.0755	35.3485	78.2449	24.3796	43.3906	2.13465952	93
18.8278	73.6539	37.4908	76.7573	2.3902	42.8883	29.84734370	91
35.3485	73.3993	32.4234	75.5156	31.0353	42.7069	26.20918771	91
19.2810	73.3452	2.3071	75.0439	27.8361	42.1390	35.36087443	89
32.4234	72.7873	30.4871	74.9975	34.0873	41.9818	4.37191456	87
0.1236	70.0460	28.2623	74.0611	32.7635	40.7946	48.87780781	76

The frequency resolution is $1/(27,19666 \text{ d}) = 0,04(\text{d}^{-1})$

Table3.1:Frequencies as given by different methods for HD 174966.

fft (inpainting interpolated)		fft(MIARMA interpolated)		Direct Deconvolution		SigSpec's prew hitening	
Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude
0.0412	386.3303	5.5206	166.2653	0.0368	542.1340	0.04950000	0.1506
5.5206	164.2421	12.4832	330.2049	0.0735	239.4781	0.04950000	0.0878
12.4832	323.7297	15.5731	148.2804	0.9927	258.1764	0.07030513	0.1720
15.5731	158.3368	17.5919	234.1499	3.7504	167.8468	0.10569453	0.1694
17.5919	238.5730	17.6331	749.4493	4.7431	327.3301	0.13230194	0.0782
17.6331	712.3571	17.6743	171.0959	8.1993	166.1239	0.16338575	0.0422
18.1274	278.7660	18.1274	306.2795	8.2361	256.7793	0.21222393	0.0463
19.6106	166.7930	19.6106	179.0570	10.2215	331.0992	0.24471237	0.0284
21.3821	181.6359	21.3821	159.9660	11.9864	269.6772	0.36376517	0.0302
21.4233	2765.4618	21.4233	2895.9376	13.9719	216.8562	0.58249741	0.0320
21.4645	157.1398	21.4645	170.5083	14.0087	186.5848	0.75789031	0.0591
23.1949	8340.5675	23.1949	8678.0715	14.9646	192.3635	1.77854494	0.0783
26.4496	152.9917	26.2436	158.9426	15.9574	188.1674	3.63662438	0.0313
26.4908	165.0871	26.3672	153.3780	17.6119	353.1830	5.53344382	0.1406
26.5320	181.9189	26.4084	167.7864	17.7223	180.3729	5.57944634	0.0469
26.5732	203.1810	26.4496	171.3030	18.7150	286.8635	6.30246373	0.0748
26.6144	223.1734	26.4908	184.9616	19.7077	194.6005	6.91300419	0.0687
26.6556	252.7571	26.5320	217.7256	21.1785	262.7249	9.59168569	0.0312
26.6968	295.2816	26.5732	236.1051	21.3623	256.1517	10.91275821	0.0475
26.7380	353.2153	26.6144	250.5433	21.3991	731.2734	11.91308048	0.0975
26.7792	434.2959	26.6556	272.7034	21.4358	1091.2148	12.47882154	0.2462
26.8204	566.1395	26.6968	331.2326	21.4726	302.2405	15.57686071	0.1238
26.8616	817.5011	26.7380	392.9852	21.5094	198.8218	15.91208339	0.1057
26.9028	1426.5343	26.7792	487.0506	23.0904	223.0284	16.21017689	0.1002
26.9440	5481.3981	26.8204	619.1059	23.1272	332.9741	17.40844864	0.0432
26.9852	2990.0312	26.8616	873.1195	23.1639	777.6656	17.62250520	0.6038
27.0264	1180.3270	26.9028	1518.6526	23.2007	4130.7038	18.13525291	0.2308
27.0676	737.5726	26.9440	5756.4537	23.2375	570.9945	19.04660919	0.0978
27.1088	537.5100	26.9852	3116.0139	23.2742	316.6635	19.62083260	0.1361
27.1500	424.7397	27.0264	1219.1612	23.3110	218.8052	20.62812578	0.0442
27.1912	352.9623	27.0676	747.9100	23.3478	178.1600	20.82386719	0.0379
27.2324	303.2588	27.1088	543.3436	24.9656	170.9330	21.22309483	0.0821
27.2736	267.6108	27.1500	421.9279	26.8040	174.4364	21.42080448	20929
27.3148	234.0444	27.1912	323.3979	26.8408	219.6296	23.19481516	62902
27.3560	212.8112	27.2324	291.4421	26.8775	320.3173	23.19771104	0.0631
27.3972	199.3601	27.2736	244.4242	26.9143	603.9608	23.68167050	0.0562
27.4384	186.8939	27.3148	200.7722	26.9511	3265.8069	23.91587645	0.0338
27.4796	177.3867	27.3560	169.2305	26.9878	825.9371	24.30838825	0.0398
27.5208	180.5925	27.3972	157.6096	27.0246	359.1559	25.09353708	0.0429
27.5620	169.5593	27.6855	349.9165	27.0614	227.6290	26.95851157	51034
27.6031	186.9410	27.7267	1323.1465	27.0981	167.5896	26.96164077	0.0312
27.6443	240.0685	27.7679	359.3772	27.7232	646.0948	27.71548633	0.9973
27.6855	473.9435	27.8091	233.5478	29.9293	216.0799	29.12558839	0.0333
27.7267	1152.9799	27.8503	181.3946	31.6942	225.5358	30.22708013	0.0376
27.7679	253.7725	27.8915	172.3104	33.6796	245.3053	30.95009515	0.1157
27.8091	152.6503	27.9327	151.2469	42.9084	180.8416	32.30995611	0.0329
44.6182	173.5641	44.6182	191.5327	46.6588	308.6810	42.24100060	0.0294
50.1389	349.8391	50.1389	385.2399	50.1518	226.4780	44.61829076	0.1301
50.1801	188.8735	50.1801	189.5763	61.6234	211.4256	50.15324453	0.3111
50.9216	173.6153	50.9216	200.9587	74.6026	176.9140	50.91071289	0.1497

The frequency resolution is $1/(27,19666 \text{ d}) = 0,04(\text{d}^{-1})$

Table3.2:Frequencies as given by different methods for HD 174966 (amplitude ordered).

fft (inpainting interpolated)		fft(MIARMA interpolated)		Direct Deconvolution		SigSpec's prewhitening	
Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude	Frequency(d ⁻¹)	Amplitude
23.1949	8340.5675	23.1949	8678.0715	23.2007	4130.7038	23.19481516	62902
26.9440	5481.3981	26.9440	5756.4537	26.9511	3265.8069	26.95851157	51034
26.9852	2990.0312	26.9852	3116.0139	21.4358	1091.2148	21.42080448	20929
21.4233	2765.4618	21.4233	2895.9376	26.9878	825.9371	27.71548633	0.9973
26.9028	1426.5343	26.9028	1518.6526	23.1639	777.6656	17.62250520	0.6038
27.0264	1180.3270	27.7267	1323.1465	21.3991	731.2734	50.15324453	0.3111
27.7267	1152.9799	27.0264	1219.1612	27.7232	646.0948	12.47882154	0.2462
26.8616	817.5011	26.8616	873.1195	26.9143	603.9608	18.13525291	0.2308
27.0676	737.5726	17.6331	749.4493	23.2375	570.9945	0.07030513	0.1720
17.6331	712.3571	27.0676	747.9100	0.0368	542.1340	0.10569453	0.1694
26.8204	566.1395	26.8204	619.1059	27.0246	359.1559	0.04950000	0.1506
27.1088	537.5100	27.1088	543.3436	17.6119	353.1830	50.91071289	0.1497
27.6855	473.9435	26.7792	487.0506	23.1272	332.9741	5.53344382	0.1406
26.7792	434.2959	27.1500	421.9279	10.2215	331.0992	19.62083260	0.1361
27.1500	424.7397	26.7380	392.9852	4.7431	327.3301	44.61829076	0.1301
0.0412	386.3303	50.1389	385.2399	26.8775	320.3173	15.57686071	0.1238
26.7380	353.2153	27.7679	359.3772	23.2742	316.6635	30.95009515	0.1157
27.1912	352.9623	27.6855	349.9165	46.6588	308.6810	15.91208339	0.1057
50.1389	349.8391	26.6968	331.2326	21.4726	302.2405	16.21017689	0.1002
12.4832	323.7297	12.4832	330.2049	18.7150	286.8635	19.04660919	0.0978
27.2324	303.2588	27.1912	323.3979	11.9864	269.6772	11.91308048	0.0975
26.6968	295.2816	18.1274	306.2795	21.1785	262.7249	0.04950000	0.0878
18.1274	278.7660	27.2324	291.4421	0.9927	258.1764	21.22309483	0.0821
27.2736	267.6108	26.6556	272.7034	8.2361	256.7793	1.77854494	0.0783
27.7679	253.7725	26.6144	250.5433	21.3623	256.1517	0.13230194	0.0782
26.6556	252.7571	27.2736	244.4242	33.6796	245.3053	6.30246373	0.0748
27.6443	240.0685	26.5732	236.1051	0.0735	239.4781	6.91300419	0.0687
17.5919	238.5730	17.5919	234.1499	27.0614	227.6290	23.19771104	0.0631
27.3148	234.0444	27.8091	233.5478	50.1518	226.4780	0.75789031	0.0591
26.6144	223.1734	26.5320	217.7256	31.6942	225.5358	23.68167050	0.0562
27.3560	212.8112	50.9216	200.9587	23.0904	223.0284	10.91275821	0.0475
26.5732	203.1810	27.3148	200.7722	26.8408	219.6296	5.57944634	0.0469
27.3972	199.3601	44.6182	191.5327	23.3110	218.8052	0.21222393	0.0463
50.1801	188.8735	50.1801	189.5763	13.9719	216.8562	20.62812578	0.0442
27.6031	186.9410	26.4908	184.9616	29.9293	216.0799	17.40844864	0.0432
27.4384	186.8939	27.8503	181.3946	61.6234	211.4256	25.09353708	0.0429
26.5320	181.9189	19.6106	179.0570	21.5094	198.8218	0.16338575	0.0422
21.3821	181.6359	27.8915	172.3104	19.7077	194.6005	24.30838825	0.0398
27.5208	180.5925	26.4496	171.3030	14.9646	192.3635	20.82386719	0.0379
27.4796	177.3867	17.6743	171.0959	15.9574	188.1674	30.22708013	0.0376
50.9216	173.6153	21.4645	170.5083	14.0087	186.5848	23.91587645	0.0338
44.6182	173.5641	27.3560	169.2305	42.9084	180.8416	29.12558839	0.0333
27.5620	169.5593	26.4084	167.7864	17.7223	180.3729	32.30995611	0.0329
19.6106	166.7930	5.5206	166.2653	23.3478	178.1600	0.58249741	0.0320
26.4908	165.0871	21.3821	159.9660	74.6026	176.9140	3.63662438	0.0313
5.5206	164.2421	26.2436	158.9426	26.8040	174.4364	26.96164077	0.0312
15.5731	158.3368	27.3972	157.6096	24.9656	170.9330	9.59168569	0.0312
21.4645	157.1398	26.3672	153.3780	3.7504	167.8468	0.36376517	0.0302
26.4496	152.9917	27.9327	151.2469	27.0981	167.5896	42.24100060	0.0294
27.8091	152.6503	15.5731	148.2804	8.1993	166.1239	0.24471237	0.0284

The frequency resolution is $1/(27,19666 \text{ d}) = 0,04(\text{d}^{-1})$

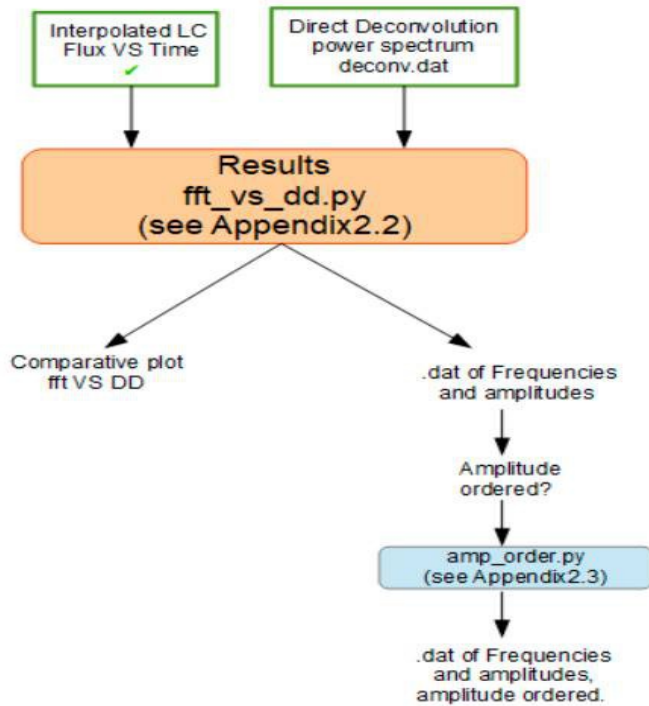
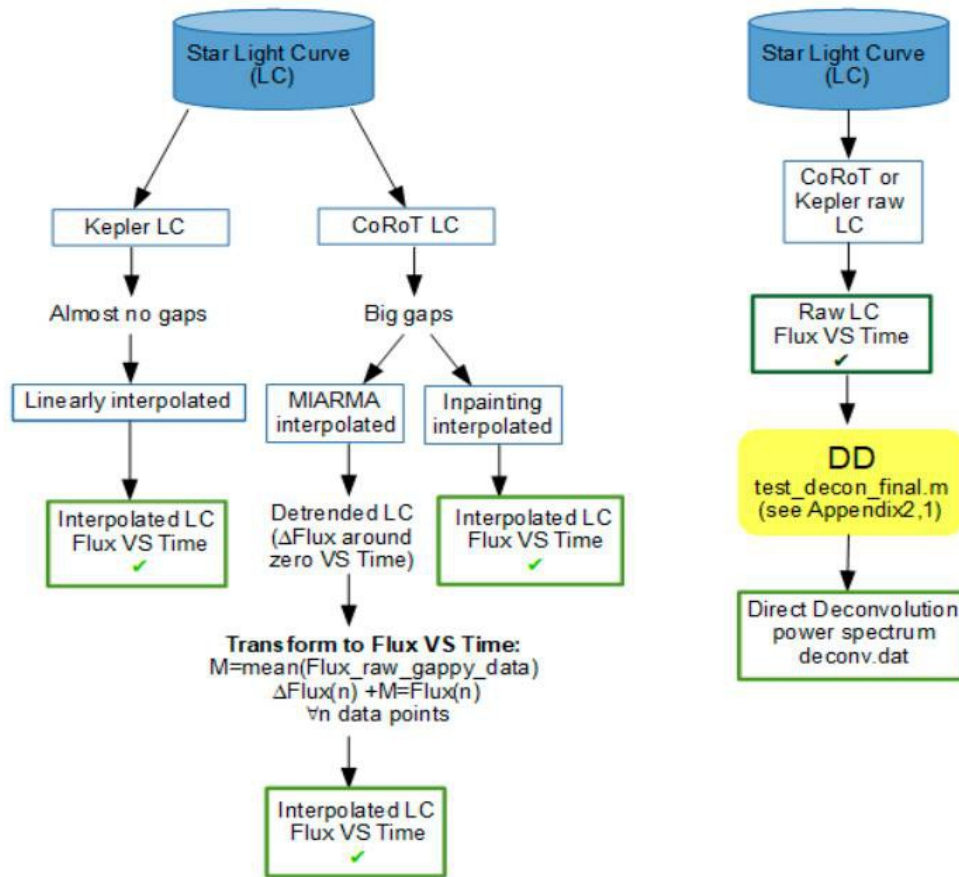
Section three:Conclusions so far.

- Deviations from the original cadence are very frequent, even a very small deviation can lead to an error in our interpretation of the excited frequencies. Direct deconvolution could be very helpful in this kind of situations as it assumes unevenly space data.

Section four:Open Issues and upcoming results.

- The frequency limit ($f_{\text{máx}}$)
- The fundamental frequency (f_0)
- Conclusions from CoRoT data are in progress
- Test the direct deconvolution with solar light curve and other types of stars.
- Search models in TUCAN for these new frequencies product of the direct deconvolution treatment instead of a prewhitening.
- Refactoring of the scrips.

Appendix1: Pipeline:



Appendix2.1: Test_deconv_final.m

```
% test_decon_final.m
% Test Fourier deconvolution on synthetic, unevenly spaced data with gaps

q = 1;
p = 1;
two_pi = 2 * pi;

xx = importdata('name of the light curve file.dat');
tt_0 = xx(:,1);
nx = length(tt_0)/p;
tt = xx(1:nx,1)';
counts = xx(1:nx,2)';
xx = counts-mean(counts);
nsp_1 = nx;
nsp = 2^floor(log(nsp_1)/log(2))

xx_window = ones( size( xx ) );

% Define frequency array ww_vec (radians/unit time)
wfac = 1 - ( 1 / nx );
tt_range = max( tt ) - min( tt );
ww_fund = wfac * two_pi / tt_range; % Fundamental frequency

freq_extend = 1;% if > 1 extend frequency range to higher frequencies
num_ww = freq_extend * nsp;
ww_use = 0: ww_fund: (num_ww - 1) * ww_fund; % Fundamental + harmonics

    xx_use = xx;
    xx_window_use = xx_window;
    tt_use = tt;

%% compute Fourier transform of arbitrarily spaced data
%% Modified from fortran code in ApJ 343, 1989, 874-887, Paper III

clear data_in
data_in.xx_vec = xx_use;
data_in.tt_vec = tt_use;
data_in.ww_vec = ww_use;
data_in.ft_sign = 1;
data_in.tt_zero = tt_use(1);
data_out = ft_uneven( data_in );
xx_ft = data_out.ft_vec;% FT of data
xx_ls = data_out.ls_vec;% L-S of data
%sp_gap = nx*abs( xx_ft(1:nsp) ) .^ 2;
sp_ls = nx*abs( xx_ft(1:nsp) ) .^ 2;

clear data_in
data_in.xx_vec = xx_window_use;
data_in.tt_vec = tt_use;
data_in.ww_vec = ww_use;
data_in.ft_sign = 1;
data_in.tt_zero = tt_use(1);
data_out = ft_uneven( data_in );
window_ft = data_out.ft_vec;% FT of window
window_ls = data_out.ls_vec;% L-S of window
sp_win = nx*abs( xx_ft(1:nsp) ) .^ 2;

%Decovolve FT of window from the FT of the data

e = 1e-3;%controla el power: mayor e -> menor power
zz = ifft( fft( xx_ft ) ./ (fft( window_ft )+e) ); %Direct Deconvolution
zz_final = abs(zz);

ww_use = ww_use/two_pi;

lis_zz = [ww_use', zz'];
lis_zz_final = [ww_use', zz_final'];

save('deconv_0.dat', 'lis_zz', '-ascii'); % Deconv
save('deconv.dat', 'lis_zz_final', '-ascii'); % Real part of Deconv
```

Appendix2.2: fft_vs_dd.py

```
import numpy as np
import sys
import matplotlib.pyplot as plt

s_file = str(sys.argv[1])

x_t, xx = np.genfromtxt(s_file, unpack=True)
frq2, y2 = np.genfromtxt("deconv.dat", unpack=True)

y2_n = [-1*(abs(y2[i])) for i in range(len(y2))]

x = []

am = np.mean(xx)
for i in xx:
    x.append(i-am)

q=1

nx=len(x) # length of the signal
print "# de puntos n=",nx
m = 2**int((np.log10(nx)/(np.log10(2))))
print "(m=2^n),m=",m
Fs = 1/(58.5/86400) # sampling rate corot = 32seg, Kepler_sc=60seg, Kepler_lc=1800seg, 58,5 Bowman
Fny = Fs/2 #Nyquist Frequency.

xx_m=x[:m]

fft = np.fft.rfft(xx_m)*2./(m) # fft computing and normalization
fft_freq = np.arange(0,Fny,Fs/(m))
fft_freq = fft_freq[range(len(fft)/2)]# one side frequency range
fft = fft[range(len(fft_freq))]

#-----Important Information-----

print x_t[-1]-x_t[0], "dias de obs"
print "Fny=",Fny, "d^-1"
print "Paso en frecuencias=",Fs/(m)
print "N de freq =",len(fft_freq)
print "N de fft =",len(fft)
print "N de deconv=", len(frq2)
print Fny/(Fs/(m)), m/2

#-----Frequency selection-----

#relacion senal ruido

senal_fft = max(abs(fft))
senal_dd = max(y2)
ruido_fft = 2.09039570659 #rms of the noisy section of the spectrum
ruido_dd = 1.12567766568 #rms of the noisy section of the spectrum

s_to_n_fft = np.sqrt(senal_fft/ruido_fft)
s_to_n_dd = np.sqrt(senal_dd/ruido_dd)
print "s/n fft", s_to_n_fft
print "s/n dd", s_to_n_dd

real_fft = [abs(fft[i]) for i in range(len(fft))]

y2_new = [y2[i] for i in range(len(y2)) if y2[i]>= s_to_n_dd]
fft_new = [abs(fft[i]) for i in range(len(fft)) if abs(fft[i])>= s_to_n_fft]

y2_sorted = np.sort(y2_new)
fft_sorted= np.sort(fft_new)

y2_3 = y2_sorted[-1:-51:-1]
fft_3 = fft_sorted[-1:-51:-1]
```

```

maximos_deconv = np.array([frq2[np.where(y2==y2_3[i])] for i in range(len(y2_3))])
maximos_deconv = np.sort(maximos_deconv.ravel())
maximos_deconv = np.array([[i] for i in maximos_deconv])
amp_deconv = np.array([y2[np.where(maximos_deconv[i]==frq2)] for i in range(len(maximos_deconv))])

maximos_fft = np.array([fft_freq[np.where(real_fft==fft_3[i])] for i in range(len(fft_3))])
maximos_fft = np.sort(maximos_fft.ravel())
maximos_fft = np.array([[i] for i in maximos_fft])
amp_fft = np.array([abs(fft[np.where(maximos_fft[i]==fft_freq)]) for i in range(len(maximos_fft))])

np.savetxt("maximos_dd.dat", np.hstack([maximos_deconv,amp_deconv]), fmt='%6.4f')
np.savetxt("maximos_fft.dat",np.hstack([maximos_fft,amp_fft]), fmt='%6.4f')

#-----Plot-----
fig, ax = plt.subplots(2, 1)
ax[0].plot(x_t,x,'r-')
ax[1].plot(frq2,y2_n,'b-',fft_freq,abs(fft),'r-')
ax[1].set_xlabel(r'Frequency $d^{-1}$') # plotting the spectrum

plt.show()
#-----Write fft file-----
fft_file = [[fft_freq[i],abs(fft[i])] for i in range(len(fft_freq))]
np.savetxt("fft.dat",fft_file)

```

Appendix2.3: amp_order.py

```

import numpy as np
import sys

s_file = str(sys.argv[1])
data = np.genfromtxt(s_file)

new_data = data[np.argsort(data[:,1])]
new_data = new_data[::-1]
np.savetxt('file_amp_order.dat', new_data, '%6.4f')

```